Methods & Mastery - Answers
Understanding Highest Common Factors and Lowest Common Multiples

1. Use the listing method to find the highest common factor of each pair of numbers:
   a. 12 and 20

   12: 1 × 12, 2 × 6, 3 × 4

   20: 1 × 20, 2 × 10, 4 × 5   **HCF = 4**

   b. 36 and 27

   36: 1 × 36, 2 × 18, 3 × 12, 4 × 9, 6 × 6

   27: 1 × 27, 3 × 9   **HCF = 9**

   c. 48 and 32

   48: 1 × 48, 2 × 24, 3 × 16, 4 × 12, 6 × 8

   32: 1 × 32, 2 × 16, 4 × 8   **HCF = 16**

2. Use the listing method to find the lowest common multiple of each pair of numbers:
   a. 6 and 8

   6: 6, 12, 18, 24, 30

   8: 8, 16, 24, 32, 40   **LCM = 24**

   b. 12 and 15

   12: 12, 24, 36, 48, 60

   15: 15, 30, 45, 60, 75   **LCM = 60**

   c. 8 and 18

   8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80

   18: 18, 36, 54, 72, 90, 108, 126, 144, 162, 180   **LCM = 72**

3. Use the Venn diagram method to find the highest common factor and lowest common multiple of each pair of numbers:

    a. 20 and 24

       HCF = 4

       LCM = 120

    b. 36 and 42

       HCF = 6

       LCM = 252

    c. 10 and 8

       HCF = 2

       LCM = 40

4. When do you think it is better to use the listing method to find the HCF or LCM?

    Any suitable answer.

5. When do you think it is better to use the Venn diagram method to find the HCF or LCM?

    Any suitable answer.

6. When using the listing method to find the LCM, why do you think we only list a few multiples of each number at a time?

    Any suitable answer.

7.   a. Write your own LCM or HCF question. Make the question as easy as possible.

       Any suitable question.

    b. Answer your question.

       Correct answer to 7a.

    c. Explain what you did to make your question easy. Was it as easy as you thought it would be?

       Any suitable answer.

    d. Which method did you use to solve it? Why did you choose that method?

    Any suitable answer.

8.   a. Write your own LCM or HCF question. Make the question as hard as possible.

    Any suitable question.

    b. Answer your question.

    Correct answer to 8a.

    c. What did you do to make your question difficult?

    Any suitable answer.

    d. Which method did you use to solve it? Why did you choose that method?

    Any suitable answer.

Methods & Mastery - Content Description
Understanding Highest Common Factors and Lowest Common Multiples

This content pack is aimed at teachers, tutors and home educators who would like to help their students develop a better understanding of how to find highest common factors and lowest common multiples, by encouraging students to discuss the strengths and weaknesses of different methods of working.

Two methods are discussed in this content:

1. **The listing method** – Listing out all the factors, or the first few multiples, of a pair of numbers in order to choose the highest factor common to both lists, or the lowest multiple common to both lists.

2. **The prime factor method** – Finding the prime factors of both numbers, then using a Venn diagram to find the highest common factor and lowest common multiple.

In order to allow you to choose the most convenient format, the learning material is presented in a number of different ways:

Independent Learning	This sheet gives an explanation of two methods of finding the highest common factor and lowest common multiple, with examples, followed by questions. These questions give students an opportunity to practise the methods. Written response questions also encourage students to think about which method they would choose for a problem.
Slideshow	This PowerPoint presentation contains the same content as the independent learning worksheet, arranged to be more conveniently presented to a class. The questions are split into two sets, one for practice and one for discussion, and content is arranged with a starter, plenary and opportunities for classroom discussion. While this slideshow can be used as a lesson, the content may need to be cut or extended depending on the range of ability in the class.
Worksheet (answer spaces)	This worksheet contains the questions from the slideshow in a printable format. It has space for students to answer on the sheet.
Worksheet (no answer spaces)	This worksheet contains the questions from the slideshow in a printable format. To save printing, it has no spaces for students to answer on the sheet.
Plenary	Two questions to develop discussion at the end of a lesson. Each question gives an example of one of the two methods discussed and asks for an advantage and disadvantage of that method.

Methods & Mastery - Independent Learning
Understanding Highest Common Factors and Lowest Common Multiples

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Prior Knowledge:

Prime numbers and prime factorisation.
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A factor is a number that divides into another number. When you have two numbers, a common factor divides into both, without a remainder. The highest common factor (HCF) of two numbers is therefore the largest out of these common factors.

For example, the highest common factor of 6 and 8 is 2.

A multiple is a number that is in the times table of another number. When you have two numbers, a common multiple is in both of their times tables. The lowest common multiple (LCM) of two numbers is therefore the smallest of these common multiples.

For example, the lowest common multiple of 4 and 5 is 20.

Here are two methods for finding the HCF and LCM.

Method 1 (listing method)

Example 1: Find the highest common factor of 12 and 20.

List all the factors of 12 and 20.

There are a number of different techniques to listing factors. Whichever technique you use, to make sure you don't miss any out, it is important to make sure you write them down in pairs:

$$12: \quad 1 \times 12$$
$$2 \times 6$$
$$3 \times \textbf{4}$$
$$20: \quad 1 \times 20$$
$$2 \times 10$$
$$\textbf{4} \times 5$$

The highest common factor is the largest number that can be found in both lists.

The highest common factor of 12 and 20 is **4**.

Example 2: Find the lowest common multiple of 6 and 8.

List the first multiples of 6 and 8, a few multiples of each at a time.

      6:      6, 12, 18, **24**

      8:      8, 16, **24**, 32

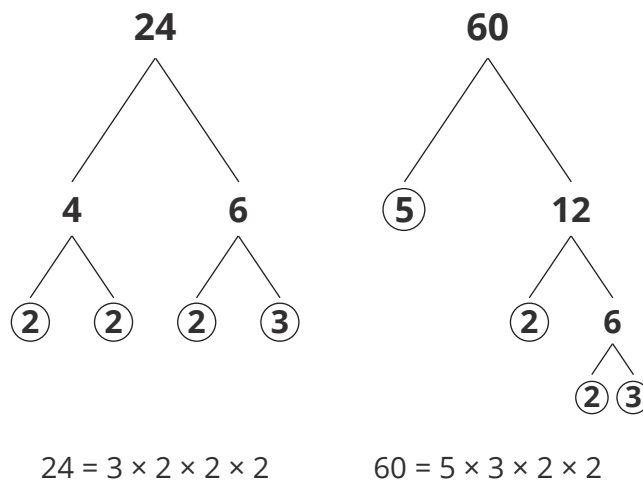The lowest common multiple is the smallest number that can be found in both lists.

The lowest common multiple of 6 and 8 is **24**.

Method 2 (prime factor method)

Example 3: Find the highest common factor and lowest common multiple of 24 and 60.

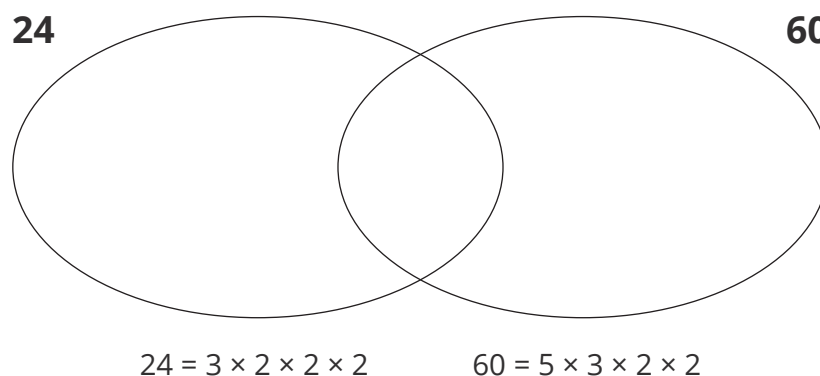Start by finding the prime factors of each number using a factor tree:

(Remember: a prime number is one that has exactly two factors – 1 and itself. The first few prime numbers are 2, 3, 5, 7, 11, …)

24

4   6

② ②   ② ③

60

⑤   12

②   6

② ③

24 = 3 × 2 × 2 × 2   60 = 5 × 3 × 2 × 2

Draw a Venn diagram (two overlapping ovals or circles). Label the left oval 24 and the right oval 60.

The prime factors of 24 will go in the left oval and the prime factors of 60 will go in the right oval. The prime factors which are common to both will go in the middle (also called the intersection).

Below each oval, write down the prime factors of each number, in order. Cross out each prime factor as you go:

24   **60**

24 = 3 × 2 × 2 × 2   60 = 5 × 3 × 2 × 2

Starting with the prime factors of 24:

- 3: This is in both lists. Write 3 in the centre, then cross out number 3 from both lists.
- 2: The first 2 is in both lists. Write 2 in the centre, then cross out a single 2 from both lists.
- 2: The second 2 is in both lists. Write 2 in the centre, then cross out another 2 from both lists.
- 2: The third 2 is only a prime factor of 24 – all the 2s in the prime factors of 60 have been crossed out. Write 2 in the left oval (for 24), then cross out the final 2 in the prime factors of 24.

Moving on to the prime factors of 60:

- 5: This is the only prime factor not crossed out. 5 is not a prime factor of 24 so write 5 in the right oval (for 60), then cross it out.

$$24 = \cancel{3} \times \cancel{2} \times \cancel{2} \times \cancel{2} \qquad 60 = \cancel{5} \times \cancel{3} \times \cancel{2} \times \cancel{2}$$
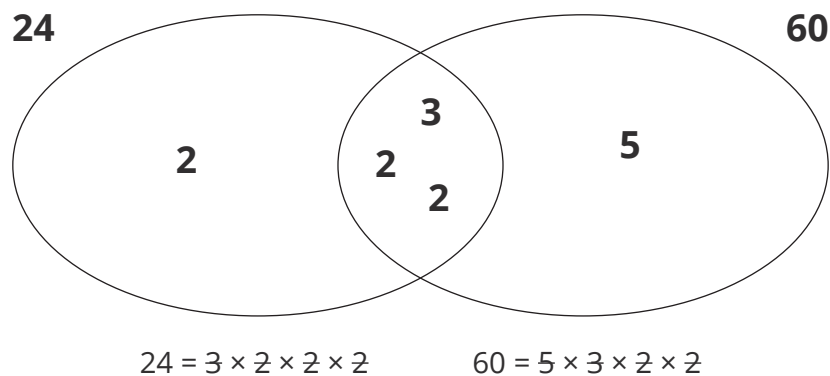
Finally, to find the HCF and LCM:

HCF: Multiply all the numbers in the **middle** of the Venn diagram:

$2 \times 2 \times 3 = 12$

The highest common factor of 24 and 60 is **12**.

LCM: Multiply **all the numbers** in the Venn diagram:

$2 \times 2 \times 3 \times 2 \times 5 = 120$

The lowest common multiple of 24 and 60 is **120**.

Your Turn:

1. Use the listing method to find the highest common factor of each pair of numbers:

   a. 12 and 20

   _____

   _____

   b. 36 and 27

   _____

   _____

    c. 48 and 32

2. Use the listing method to find the lowest common multiple of each pair of numbers:

    a. 6 and 8

    b. 12 and 15

    c. 8 and 18

3. Use the Venn diagram method to find the highest common factor and lowest common multiple of each pair of numbers:

    a. 20 and 24

    b. 36 and 42

    c. 10 and 8

4. When do you think it is better to use the listing method to find the HCF or LCM?

_____

_____

5. When do you think it is better to use the Venn diagram method to find the HCF or LCM?

_____

_____

6. When using the listing method to find the LCM, why do you think we only list a few multiples of each number at a time?

_____

_____

7.   a. Write your own LCM or HCF question. Make the question as easy as possible.

    _____

    _____

  b. Answer your question.

c. Explain what you did to make your question easy. Was it as easy as you thought it would be?

_____

_____

d. Which method did you use to solve it? Why did you choose that method?

_____

_____

8. a. Write your own LCM or HCF question. Make the question as hard as possible.

_____

_____

b. Answer your question.

c. What did you do to make your question difficult?

_____

_____

d. Which method did you use to solve it? Why did you choose that method?

_____

_____

Methods & Mastery - Plenaries
Understanding Highest Common Factors and Lowest Common Multiples

Plenary 1:

In pairs, each try to pick the easiest pair of numbers from which to find the highest common factor. Then try to pick a hard pair of numbers.

Swap questions with your partner and try to answer each other's questions. For each of the four questions in your pair, write a sentence explaining what made it easy or hard.

Plenary 2:

In pairs, each try to pick the easiest pair of numbers from which to find the lowest common multiple. Then try to pick a hard pair of numbers.

Swap questions with your partner and try to answer each other's questions. For each of the four questions in your pair, write a sentence explaining what made it easy or hard.

Methods & Mastery

Understanding Highest Common Factors and Lowest Common Multiples
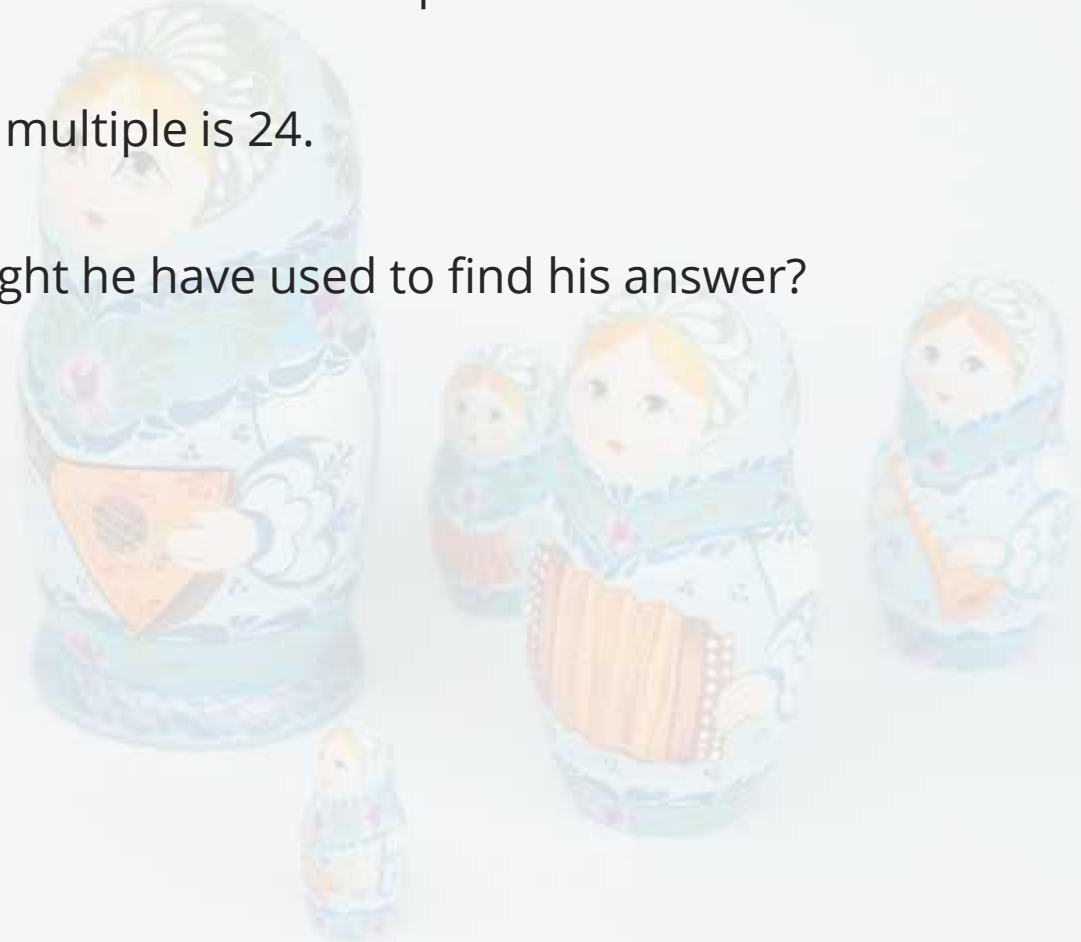
Starter

Sadiq is trying to find the lowest common multiple of 4 and 6.

He says the lowest common multiple is 24.

Is he right? What method might he have used to find his answer?

A **factor** is a number that divides into another number without a remainder. When you have two numbers, a common factor divides into both. The **highest common factor** (HCF) of two or more numbers is the biggest number that will divide into both of them.

For example, the highest common factor of 6 and 8 is 2.

A **multiple** is a number that is in the times tables of another number. When you have two numbers, a common multiple is in both of their times tables. The **lowest common multiple** (LCM) of two or more numbers is the smallest number that is in both numbers' times tables.

For example, the lowest common multiple of 4 and 5 is 20.

Here are two methods for finding the HCF and LCM.

Method 1 (Listing Method)

Example 1: Find the highest common factor of 12 and 20.

List all the factors of 12 and 20.

There are a number of different techniques to listing factors. Whichever technique you use, to make sure you don't miss any out, it is important to make sure you write them down in pairs:

12:       1 × 12   20:   1 × 20

            2 × 6   2 × 10

            3 ×④   ④× 5

The highest common factor is the largest number that can be found in both lists.

The highest common factor of 12 and 20 is 4.

Method 1 (Listing Method)

Example 2: Find the lowest common multiple of 6 and 8.

List the first multiples of 6 and 8, a few multiples of each at a time.

8:          8, 16, (24), 32
6:          6, 12, 18, (24)

The lowest common multiple is the smallest number that can be found in both lists.

The lowest common multiple of 6 and 8 is 24.

Method 2 (Prime Factor Method)

Example 3: Find the highest common factor and lowest common multiple of 24 and 60.

Start by finding the prime factors of each number using a factor tree:

(**Remember:** a prime number is one that has exactly two factors – 1 and itself. The first few prime numbers are 2, 3, 5, 7, 11, …)
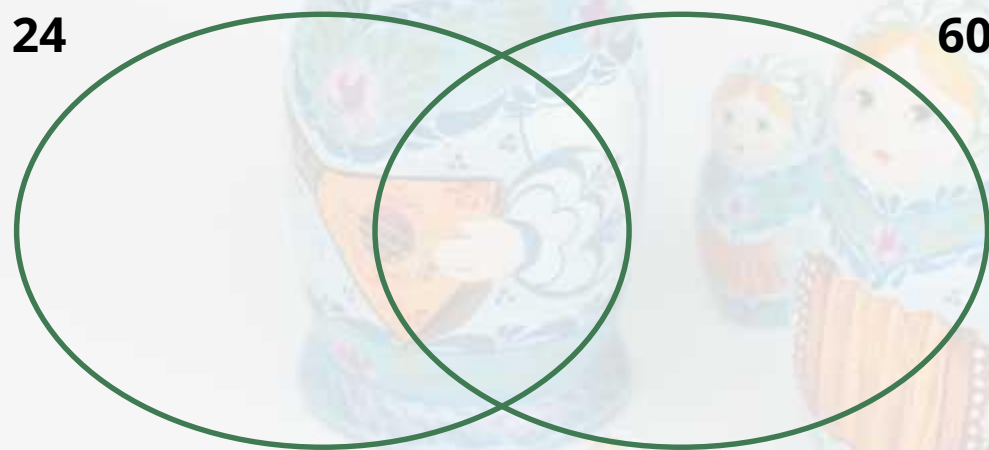

$24 = 3 × 2 × 2 × 2$ $60 = 5 × 3 × 2 × 2$

Example 3: Find the highest common factor and lowest common multiple of 24 and 60.

Draw a Venn diagram (two overlapping ovals or circles).

Label the left oval 24 and the right oval 60.

24   **60**

Example 3: Find the highest common factor and lowest common multiple of 24 and 60.

The prime factors of 24 will go in the left oval and the prime factors of 60 will go in the right oval. The prime factors which are common to both will go in the middle (also called the intersection).

Below each oval, write down the prime factors of each number, in order. Cross out each prime factor as you go:
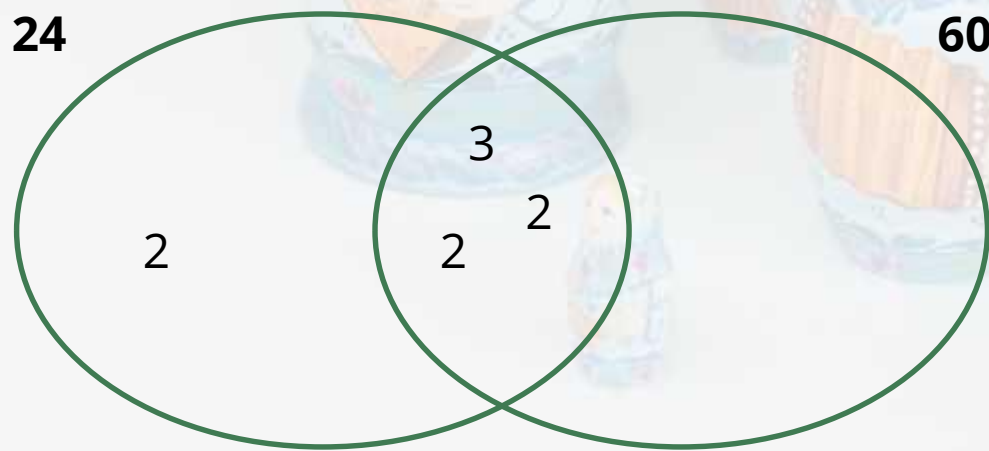
24   **60**

24 = 3 × 2 × 2 × 2   60 = 5 × 3 × 2 × 2

Example 3: Find the highest common factor and lowest common multiple of 24 and 60.

Starting with the prime factors of 24:

- 3: This is in both lists. Write 3 in the centre, then cross out number 3 from both lists.
- 2: The first 2 is in both lists. Write 2 in the centre, then cross out a single 2 from both lists.
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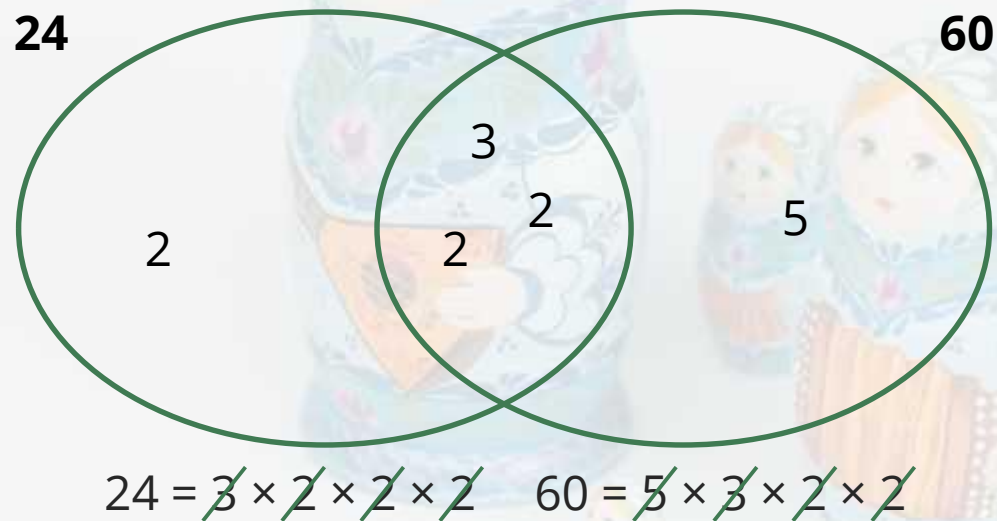
24   **60**

3

2

2   2

$24 = \cancel{3} \times \cancel{2} \times \cancel{2} \times \cancel{2}$ $60 = 5 \times \cancel{3} \times \cancel{2} \times \cancel{2}$

Example 3: Find the highest common factor and lowest common multiple of 24 and 60.

Moving on to the prime factors of 60:

- 5: This is the only prime factor not crossed out. 5 is not a prime factor of 24 so write 5 in the right oval (for 60), then cross it out.

24   **60**

2   3   2   5
         2

$24 = 3 \times 2 \times 2 \times 2 \qquad 60 = 5 \times 3 \times 2 \times 2$

Example 3: Find the highest common factor and lowest common multiple of 24 and 60.

Finally, to find the HCF and LCM:

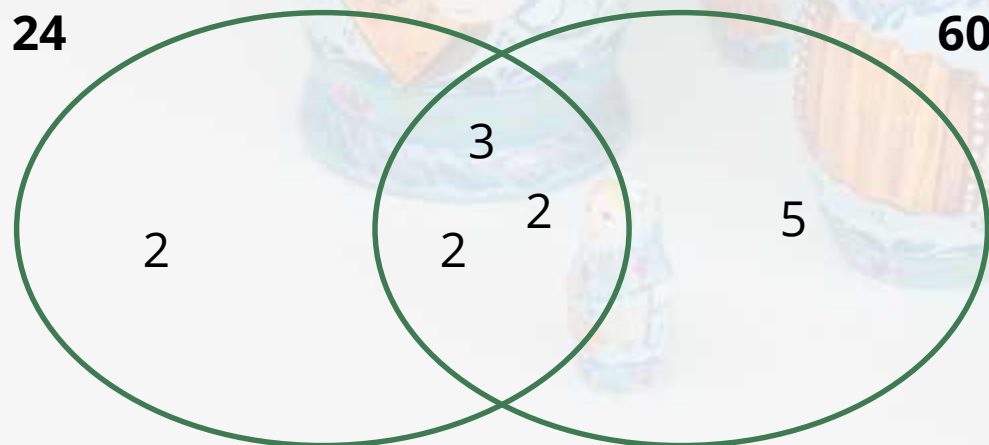HCF: Multiply all the numbers in the middle of the Venn diagram:

$2 \times 2 \times 3 = 12$

The highest common factor of 24 and 60 is **12**.

LCM: Multiply all the numbers in the Venn diagram:

$2 \times 2 \times 3 \times 2 \times 5 = 120$

The lowest common multiple of 24 and 60 is **120**.

24   **60**

3

2

2   2   5

Your Turn

1. Use the listing method to find the highest common factor of each pair of numbers:
a. 12 and 20
b. 36 and 27
c. 48 and 32

2. Use the listing method to find the lowest common multiple of each pair of numbers:
a. 6 and 8
b. 12 and 15
c. 8 and 18

3. Use the Venn diagram method to find the highest common factor and lowest common multiple of each pair of numbers:
a. 20 and 24
b. 36 and 42
c. 10 and 8

Your Turn

4. When do you think it is better to use the listing method to find the HCF or LCM?

5. When do you think it is better to use the Venn diagram method to find the HCF or LCM?

6. When using the listing method to find the LCM, why do you think we only list a few multiples of each number at a time?

7. a. Write your own LCM or HCF question. Make the question as easy as possible.
    b. Answer your question.
    c. Explain what you did to make your question easy. Was it as easy as you thought it would be?
    d. Which method did you use to solve it? Why did you choose that method?

8. a. Write your own LCM or HCF question. Make the question as hard as possible.
    b. Answer your question.
    c. What did you do to make your question difficult?
    d. Which method did you use to solve it? Why did you choose that method?

Answers

1. Use the listing method to find the highest common factor of each pair of numbers:

a. 12 and 20

   12: 1 × 12, 2 × 6, 3 × 4

   20: 1 × 20, 2 × 10, 4 × 5   **HCF = 4**

b. 36 and 27

   36: 1 × 36, 2 × 18, 3 × 12, 4 × 9, 6 × 6

   27: 1 × 27, 3 × 9   **HCF = 9**

c. 48 and 32

   48: 1 × 48, 2 × 24, 3 × 16, 4 × 12, 6 × 8

   32: 1 × 32, 2 × 16, 4 × 8   **HCF = 16**

Answers

2. Use the listing method to find the lowest common multiple of each pair of numbers:

a. 6 and 8

6: 6, 12, 18, 24, 30

8: 8, 16, 24, 32, 40   **LCM = 24**

b. 12 and 15

12: 12, 24, 36, 48, 60

15: 15, 30, 45, 60, 75   **LCM = 60**

c. 8 and 18

8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80

18: 18, 36, 54, 72, 90, 108, 126, 144, 162, 180   **LCM = 72**

Answers

3. Use the Venn diagram method to find the highest common factor and lowest common multiple of each pair of numbers:

a. 20 and 24

    HCF = 4

    LCM = 120

b. 36 and 42

    HCF = 6

    LCM = 252

c. 10 and 8

    HCF = 2

    LCM = 40

Answers

4. When do you think it is better to use the listing method to find the HCF or LCM?

   Any suitable answer.

5. When do you think it is better to use the Venn diagram method to find the HCF or LCM?

   Any suitable answer.

6. When using the listing method to find the LCM, why do you think we only list a few multiples of each number at a time?

   Any suitable answer.

Answers

7. a. Write your own LCM or HCF question. Make the question as easy as possible.
       Any suitable question.

    b. Answer your question.
       Correct answers to 7a.

    c. Explain what you did to make your question easy. Was it as easy as you thought it would be?
       Any suitable answer.

    d. Which method did you use to solve it? Why did you choose that method?
       Any suitable answer.

Answers

8. a. Write your own LCM or HCF question. Make the question as hard as possible.
      Any suitable question.

   b. Answer your question.
      Correct answers to 8a.

   c. What did you do to make your question difficult?
      Any suitable answer.

   d. Which method did you use to solve it? Why did you choose that method?
      Any suitable answer.

Plenary

Draw a cartoon to explain to someone how to find the HCF and LCM using the prime factor method.

You can assume they already know how to use a factor tree to find prime factors.

For an extra challenge, try to make your cartoon without using any words.

Methods & Mastery - Worksheet
Understanding Highest Common Factors and Lowest Common Multiples

1. Use the listing method to find the highest common factor of each pair of numbers:
   a. 12 and 20
   b. 36 and 27
   c. 48 and 32

2. Use the listing method to find the lowest common multiple of each pair of numbers:
   a. 6 and 8
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Understanding Highest Common Factors and Lowest Common Multiples

1. Use the listing method to find the highest common factor of each pair of numbers:

   a. 12 and 20

   _____

   _____

   b. 36 and 27

   _____

   _____

   c. 48 and 32

   _____

   _____

2. Use the listing method to find the lowest common multiple of each pair of numbers:

   a. 6 and 8

   _____

   _____

   b. 12 and 15

   _____

   _____

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   _____

   _____

3. Use the Venn diagram method to find the highest common factor and lowest common multiple of each pair of numbers:

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    _____

    _____

5. When do you think it is better to use the Venn diagram method to find the HCF or LCM?

    _____

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_____

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7. a. Write your own LCM or HCF question. Make the question as easy as possible.

_____

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b. Answer your question.

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_____